Longitudinal Modes (§7.6)

$$\vec{E}(x,y,z) = \vec{E}_0(x,y)\cdot e^{ik_z\cdot z}$$

Round-trip phase condition

$$e^{i2k_zL} = 1$$

$$2k_zL = 2\pi\cdot m \quad ; \; m: \text{integer}$$

$$k_z = \frac{\omega}{c}\cdot n_1 \qquad n_1: \text{effective refractive index.}$$

$$2\cdot\frac{\cancel{2\pi} f_m}{c}\cdot n_1 L = \cancel{2\pi}\cdot m$$

$$f_m = m\cdot\frac{c}{2n_1L} \qquad \text{Longitudinal modes.}$$

Mode spacing

$$\Delta f = f_m - f_{m-1} = \frac{c}{2n_1L} + m\cdot\frac{c}{2L}\cdot\frac{(-1)}{n_1^2}\cdot\frac{\partial n_1}{\partial f}\cdot\Delta f$$

$$\Rightarrow \Delta f = \frac{c}{2n_1L}\cdot\Big(1+\underbrace{\frac{f}{n_1}\frac{\partial n_1}{\partial f}}_{\uparrow \text{ material dispersion}}\Big)^{-1}$$

Or in terms of λ.

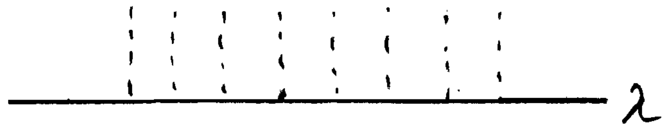
$$m\cdot\lambda = 2n_1L$$

$$\Delta\left(m = \frac{2n_1L}{\lambda}\right) \Rightarrow 1 = \frac{-1}{\lambda^2}\Delta\lambda\cdot 2n_1L + \frac{2L}{\lambda}\cdot\frac{\partial n_1}{\partial\lambda}\Delta\lambda$$

$$\Delta\lambda = \frac{-\lambda^2}{2n_1L\left(1-\frac{\lambda}{n_1}\frac{\partial n_1}{\partial\lambda}\right)}$$

Optical Spectrum

Modes

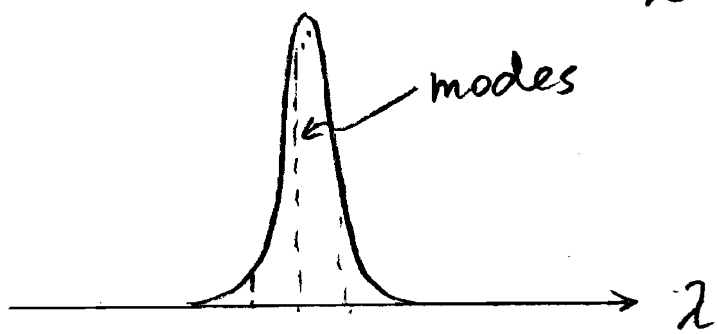


Gain

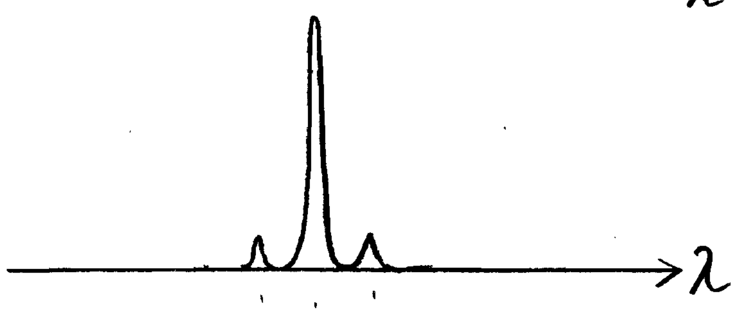
$R_1R_2\,e^{2G_n(\lambda)L}$

Amplification Factor

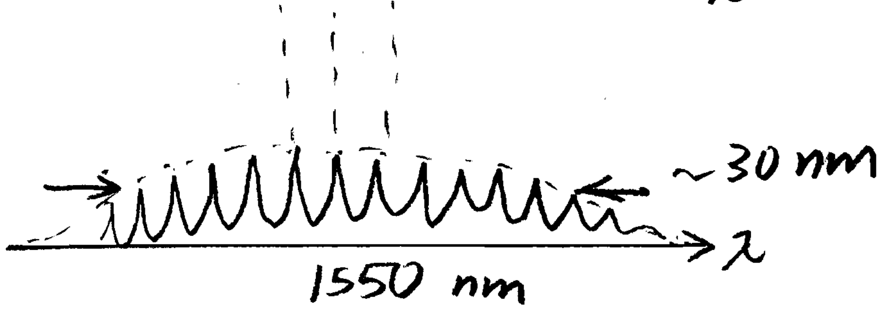
$$\frac{1}{1-R_1R_2\,e^{2G_n\cdot L}}$$



Lasing Spectrum

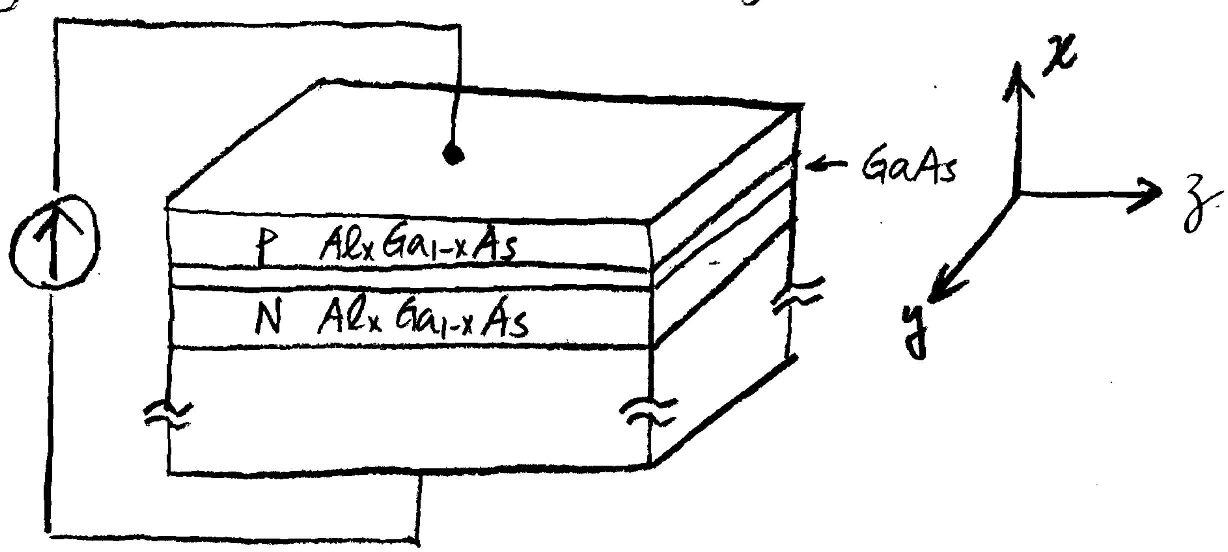


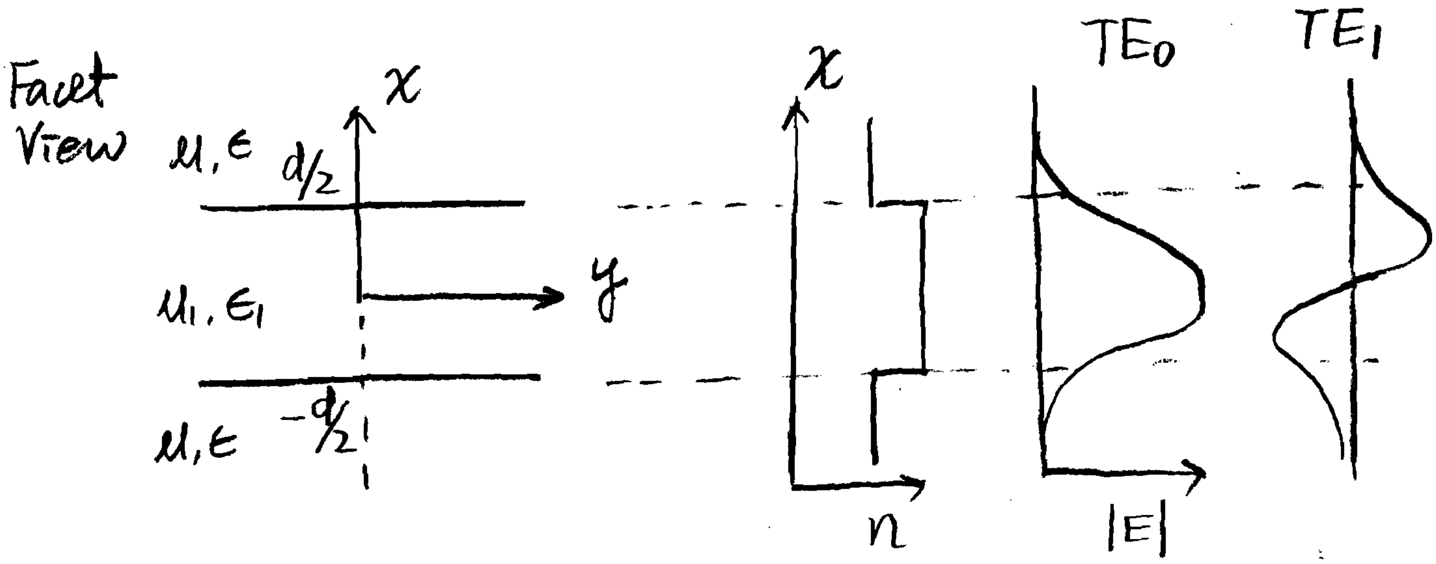
Below Threshold



Optical Waveguide

DH forms a slab waveguide





TE (Transverse Electric) Modes

H_x, H_z $\quad \vec{E} = \hat{y} E_y \quad (E_x = E_z = 0)$

TM (Transverse Magnetic) Modes

E_x, E_z $\quad \vec{H} = \hat{y} H_y \quad (H_x = H_z = 0)$

Maxwell's Eq.

$$(\nabla^2 + \omega^2 \mu \varepsilon)\vec{E} = 0$$

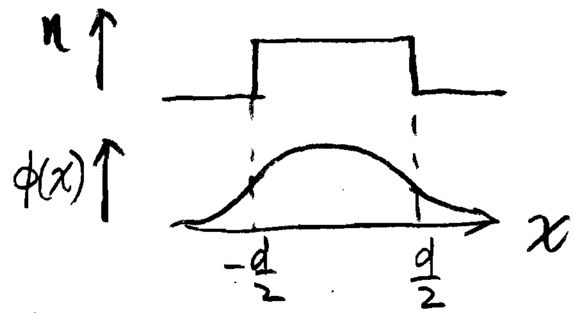
TE. $\vec{E} = \hat{y} E_y \quad (E_x = E_z = 0)$

$\frac{\partial}{\partial y} \to 0$ symmetry

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \omega^2 \mu \epsilon\right) E_y = 0$$

Similar to potential well solution

$$E_y = e^{ik_z z} \cdot \phi(x)$$



$$\phi(x) = \begin{cases} C_0 e^{-\alpha(|x| - \frac{d}{2})}, & |x| \geq \frac{d}{2} \\ C_1 \cos k_x \cdot x & |x| < \frac{d}{2} \end{cases}$$

↳ for even mode, such as TE_0, TE_2

→ $C_1 \sin k_x x$ for odd modes., TE_1, TE_3

$$\Rightarrow k_x^2 + k_z^2 = \omega^2 \mu_1 \epsilon_1 \quad \text{---} ①$$

$$-\alpha^2 + k_z^2 = \omega^2 \mu \epsilon \quad \text{---} ②$$

① − ②

$$k_x^2 + \alpha^2 = \omega^2(\mu_1 \epsilon_1 - \mu \epsilon)$$

Matching boundary coniditions

$$\begin{cases} E_x \text{ continuous} \\ H_z = \frac{1}{i\omega\mu} \cdot \frac{\partial E_y}{\partial x} \text{ continuous} \end{cases}$$

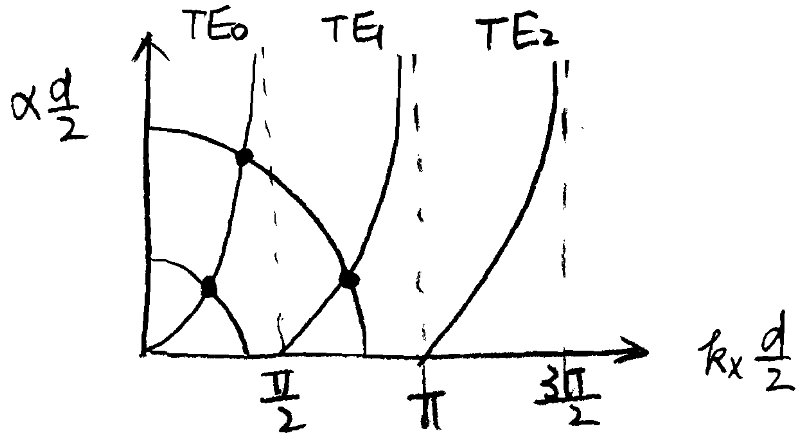
$$\Rightarrow \alpha = \frac{\mu}{\mu_1} k_x \tan\left(k_x \frac{d}{2}\right)$$

Compare with QW solution

$$\alpha = \frac{m_b}{m_w} k \tan\left(k \frac{L}{2}\right)$$

Same eigenequation!

Graphic Solution



① − ② : $k_x^2 + \alpha^2 = \omega^2 (\mu_1 \epsilon_1 - \mu \epsilon)$

$$= k_0^2 \cdot c^2 (\mu_1 \epsilon_1 - \mu \epsilon) \quad ; \quad c^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$= k_0^2 \left(\frac{\mu_1 \epsilon_1}{\mu_0 \epsilon_0} - \frac{\mu \epsilon}{\mu_0 \epsilon_0} \right)$$

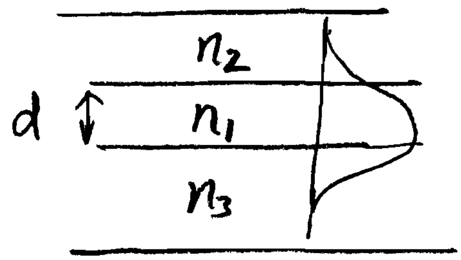
$$k_x^2 + \alpha^2 = k_0^2 (n_1^2 - n^2)$$

$$\left(k_x \frac{d}{2}\right)^2 + \left(\alpha \frac{d}{2}\right)^2 = \left(k_0 \frac{d}{2}\right)^2 (n_1^2 - n^2)$$

Single mode condition

$\alpha \frac{d}{2} \to 0$, $\quad k_x \frac{d}{2} = k_0 \frac{d}{2} \sqrt{n_1^2 - n^2} < \frac{\pi}{2}$

For general slab WG, it is customary to define 3 dimensionless parameters:



Normalized frequency $V = k_0 d \sqrt{n_1^2 - n_3^2}$

propagation parameter $b = \dfrac{n_{eff}^2 - n_3^2}{n_1^2 - n_3^2}$

asymmetry parameter $a = \dfrac{n_3^2 - n_2^2}{n_1^2 - n_3^2}$

<u>For symmetric slab</u>

$a = 0$

single mode condition: $V < \pi$

$$k_0 d \sqrt{n_1^2 - n_3^2} \approx \frac{2\pi}{\lambda} d \cdot \sqrt{\underbrace{(n_1 + n_3)}_{\approx 2n_1} \underbrace{(n_1 - n_3)}_{\approx \Delta n}} < \pi$$

$$\Rightarrow d^2 \Delta n < \frac{\lambda^2}{8 n_1}$$

Example:

$Al_{0.3}Ga_{0.7}As$	$n = 3.385$
$GaAs$	$n_1 = 3.59$
$Al_{0.3}Ga_{0.7}As$	

$> \Delta n \approx 0.205$

$\Rightarrow d < 0.36\ \mu m$ for single mode

(Vertical)
Transverse Confinement Factor

$$\Gamma = \frac{\int_{-\frac{d}{2}}^{\frac{d}{2}} |\vec{E}(x)|^2 dx}{\int_{-\infty}^{\infty} |\vec{E}(x)|^2 dx}$$

For symmetric WG with small Δn.

$$\Gamma \approx \frac{V^2}{2+V^2}$$

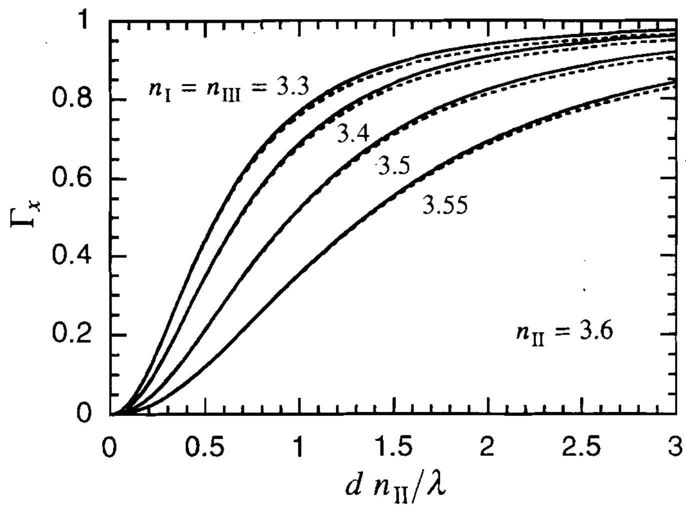
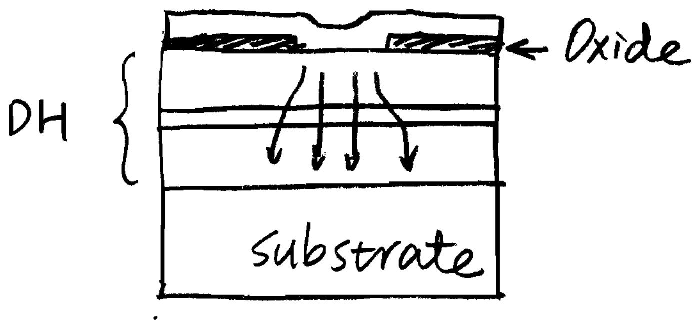


FIGURE A3.3 Comparison between the exact confinement factor (solid curve) and the approximate formula (A3.15) (dashed curve) for several values of the cladding index as a function of the guide thickness for the fundamental slab mode.

Typical Laser Structures

(cross-sectional view at facet)

Gain-guided Laser



- Easy to fabricate
- Active region wider than oxide opening due to current spreading.

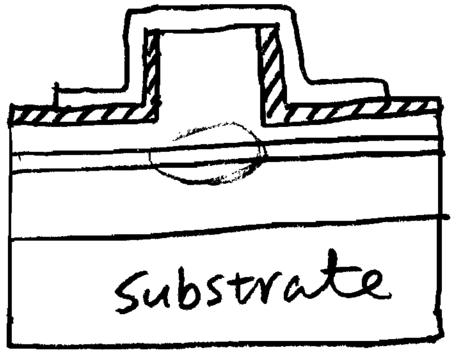
  $W_{eff} \sim 10\,\mu m$
- $I_{th} \propto W$

  → Higher threshold
- Mode

  → Easy to generate high-order lateral mode

Index-guided laser

① Ridge-waveguide laser



- Weak index guiding

  $\Delta n_{lateral} \sim 10^{-2}$
- Real index guiding

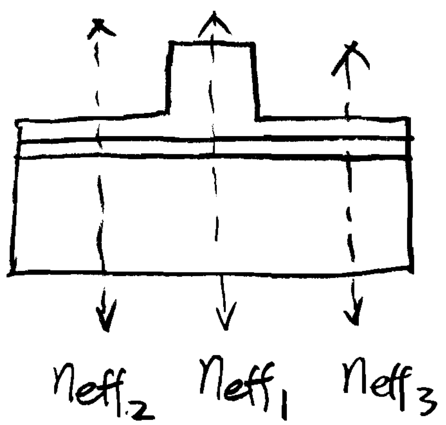
  → More stable mode
- Some current spreading
- Easy to fabricate

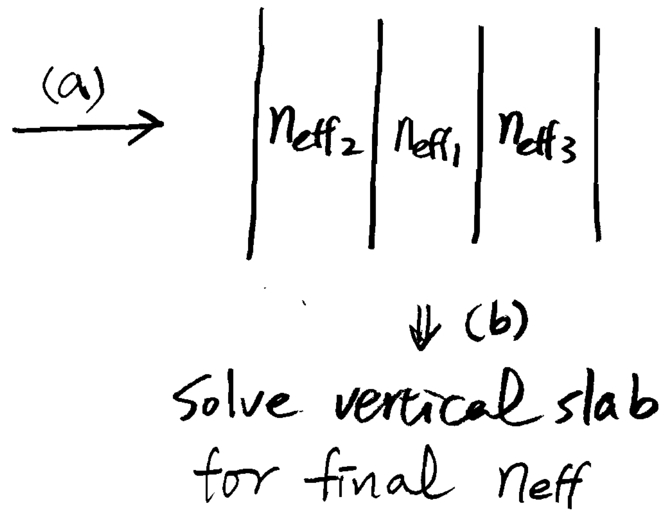
  → One epitaxial growth

Typical analysis for 2-D waveguide

- Effective Index Method

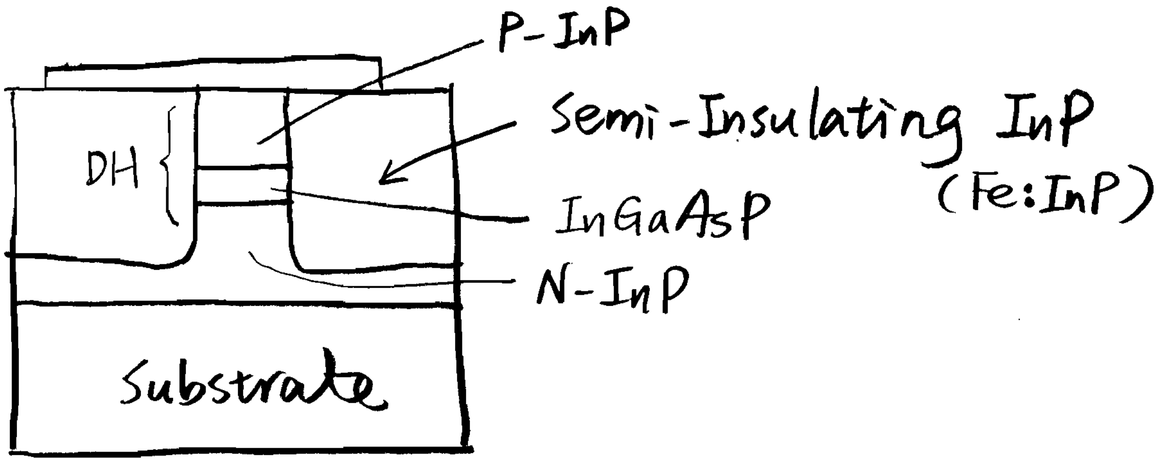
(a) Calculate n_{eff} along vertical cross-section at center and surrounding regions:





(b) Solve vertical slab waveguide for n_{eff}

② Buried Heterostructure



- Heterostructures all around active region
- No current spreading → low I_{th}
- Tight lateral guiding (larger Δn)
- Narrow laser width, $W \sim 1\,\mu m$
  → low I_{th}
- Low parasitic capacitance → higher RC bandwidth
- Require multiple epi growth (2 or 3)
- Works well for InP/InGaAsP materials